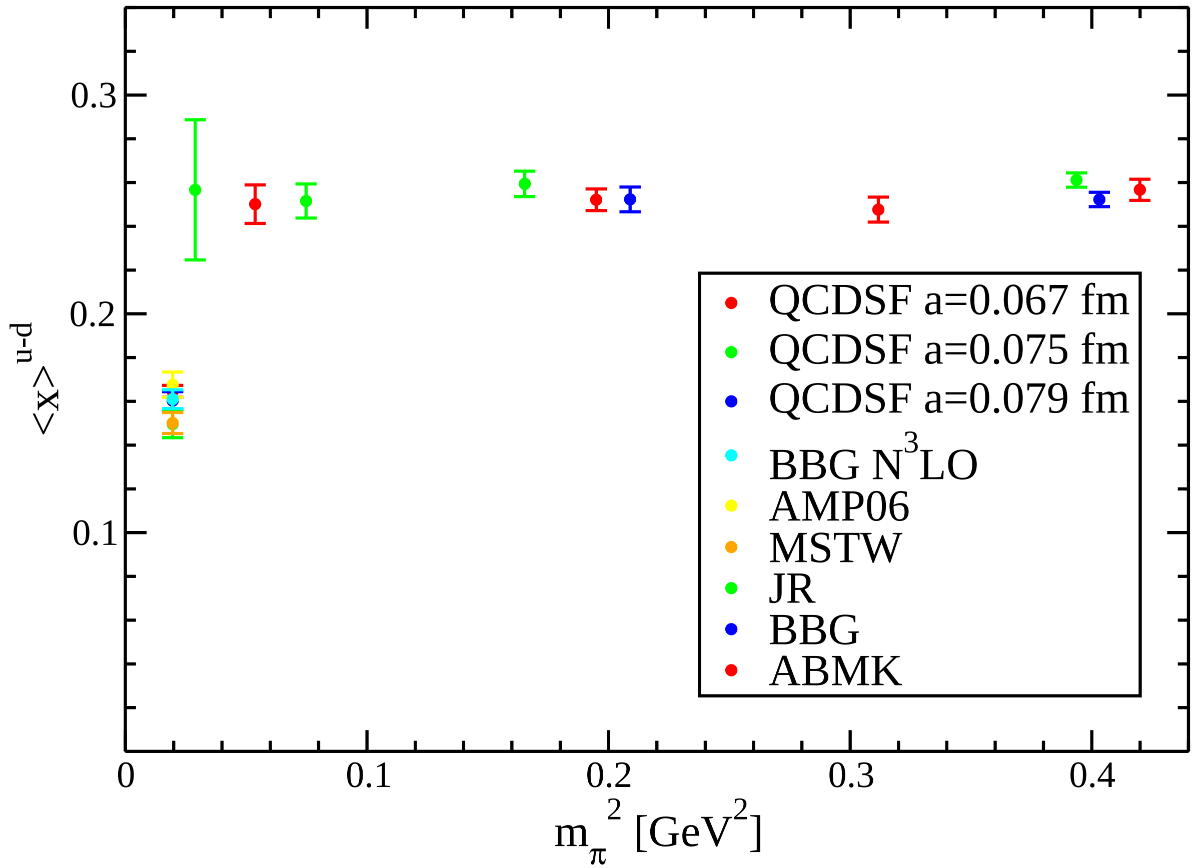
Moments of parton distribution functions from lattice QCD

Dru Renner

JLab

Provocation

- lattice calculation of $\langle x \rangle^{u-d}$ close to physical pion mass by QCDSF



QCDSF LAT2010

- there appears to be a real puzzle in the momentum fraction

Outline

- ◦ provocation
- hadronic matrix elements more broadly
- just a few details about lattice calculations of moments of PDFs
- axial coupling, lowest moment of polarized distribution
- average x, lowest non-trivial moment of unpolarized distribution
- $\langle x^2 \rangle$, next moment of unpolarized distribution
- outlook

Hadronic matrix elements

- moments of PDFs are just one component of a very big effort that
- includes form factors (FFs) and generalized parton distributions (GPDs)
- also transition FFs and transition GPDs, $N \to N^*$ and $N \to \Delta$
- not just nucleon but also pion, delta and other hadrons

Moments of parton distributions

- the x dependence in principle follows from an operator definition

$$q(x,\mu^2) = \frac{1}{2}\int\frac{dy^-}{2\pi}\, e^{ixp^+y^-}\langle p,s|\ \overline{q}(-y^-/2)\ \gamma^+\ q(y^-/2)\ |p,s\rangle\big|_{\mu^2}$$

- light-cone expansion relates moments in x,

$$\langle x^n\rangle_{q,\mu^2} = \int_{-1}^{1} dx\, x^n q(x,\mu^2) = \int_0^1 dx\, x^n \left\{q(x,\mu^2) - (-1)^n \overline{q}(x,\mu^2)\right\}$$

- to nucleon matrix elements of local operators

$$\langle p,s|\ \overline{q}\ \gamma^{\{\mu_1}\ iD^{\mu_2}\ldots iD^{\mu_n\}}q\ |p,s\rangle\big|_{\mu^2} = 2\langle x^n\rangle_{q,\mu^2}\ p^{\{\mu_1}\ldots p^{\mu_n\}}$$

- bare matrix elements and renormalization are calc. non-perturbatively

High x

- high moments n are dominated by high x (i.e. $x \to \pm 1$)

$$\langle x^n \rangle_{q,\mu^2} = \int_{-1}^{1} dx\, x^n q(x,\mu^2)$$

- the operators for low moments are multiplicatively renormalizable

$$\overline{q}\ \gamma^{\{\mu_1}\ iD^{\mu_2} \ldots iD^{\mu_n\}} q\Big|^{\overline{MS},\mu} = Z_n(\mu a)\ \ \overline{q}\ \gamma^{\{\mu_1}\ iD^{\mu_2} \ldots iD^{\mu_n\}} q\Big|^{lat,a}$$

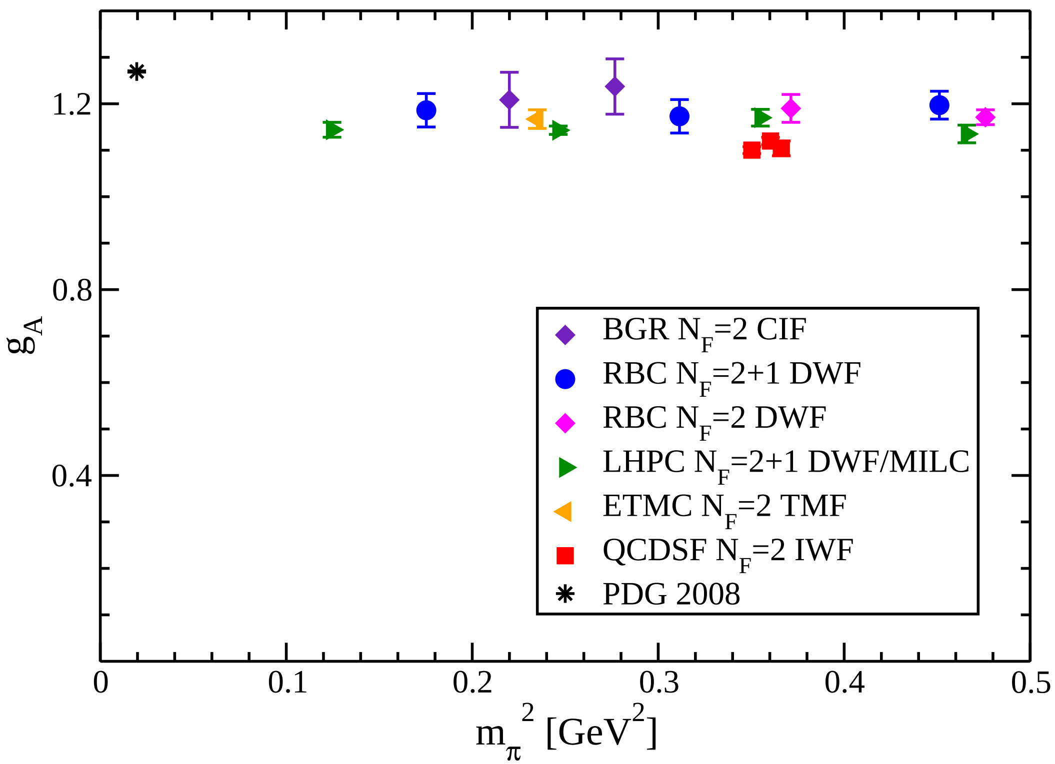
- reduced spatial symmetries allows mixing for higher moments

$$\mathcal{O}_i^{\overline{MS},\mu} = \sum_j Z_{ij}(\mu a)\ a^{-(n+2-d_j)}\ \mathcal{O}_j^{lat,a}$$

- power divergent mixing for high n and low dimension d_j

Nucleon axial coupling

- axial coupling presents one of the "better" nucleon observables

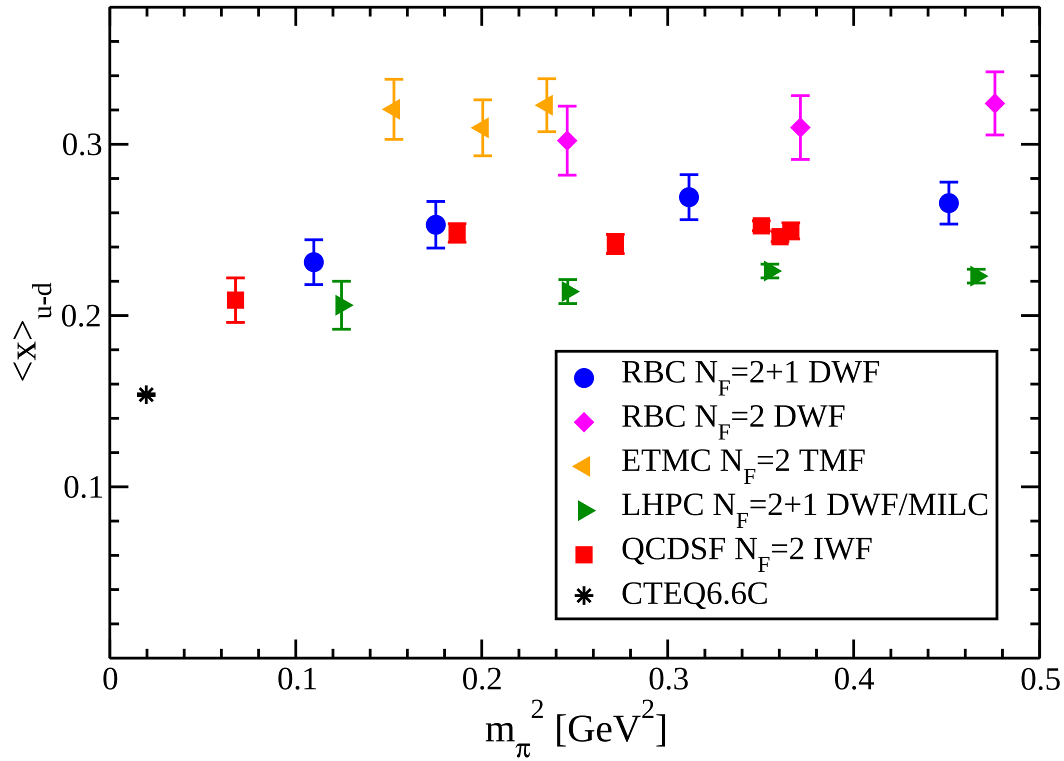


D. Renner hep-lat/1002.0925

- mild m_π dependence can still reconcile lattice and exp. result

Nucleon momentum fraction

- $\langle x \rangle$ is currently a real challenge for lattice QCD

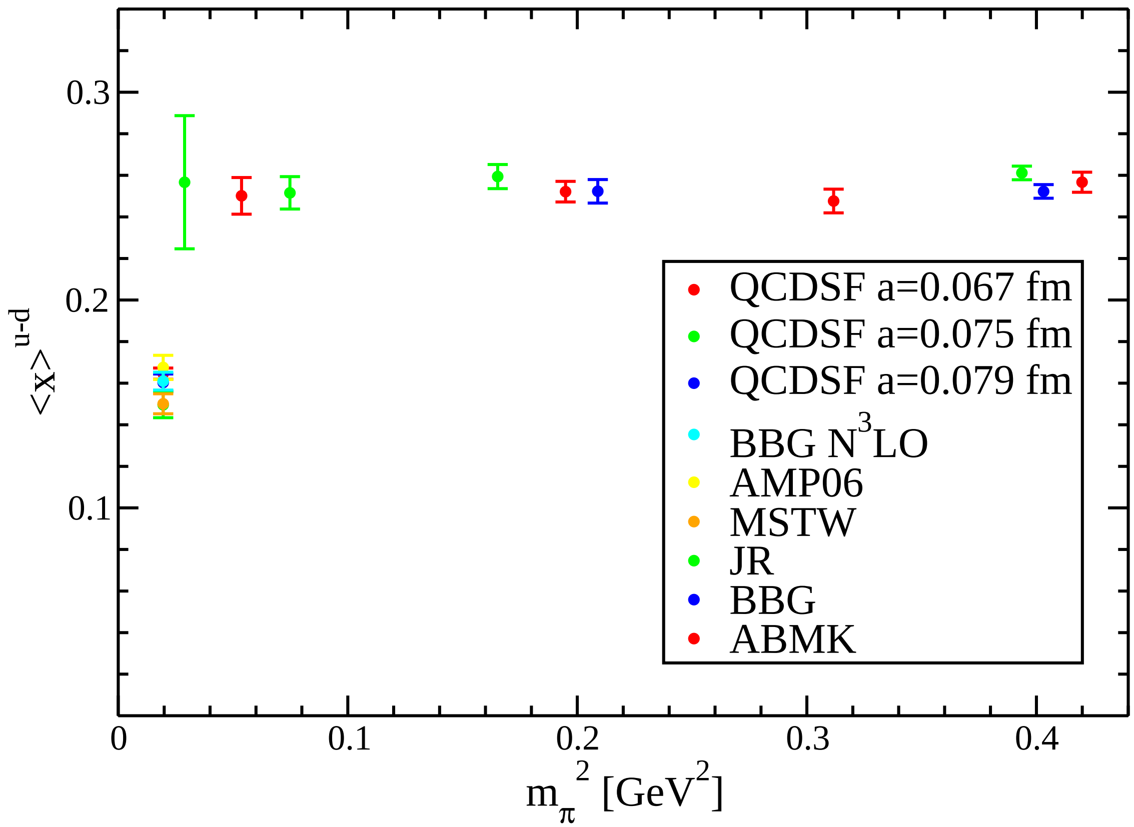


D. Renner hep-lat/1002.0925

- scatter between lattice results hints at various problems

Momentum fraction again

- flatness of $\langle x \rangle$ has been a problem since early quenched calculations

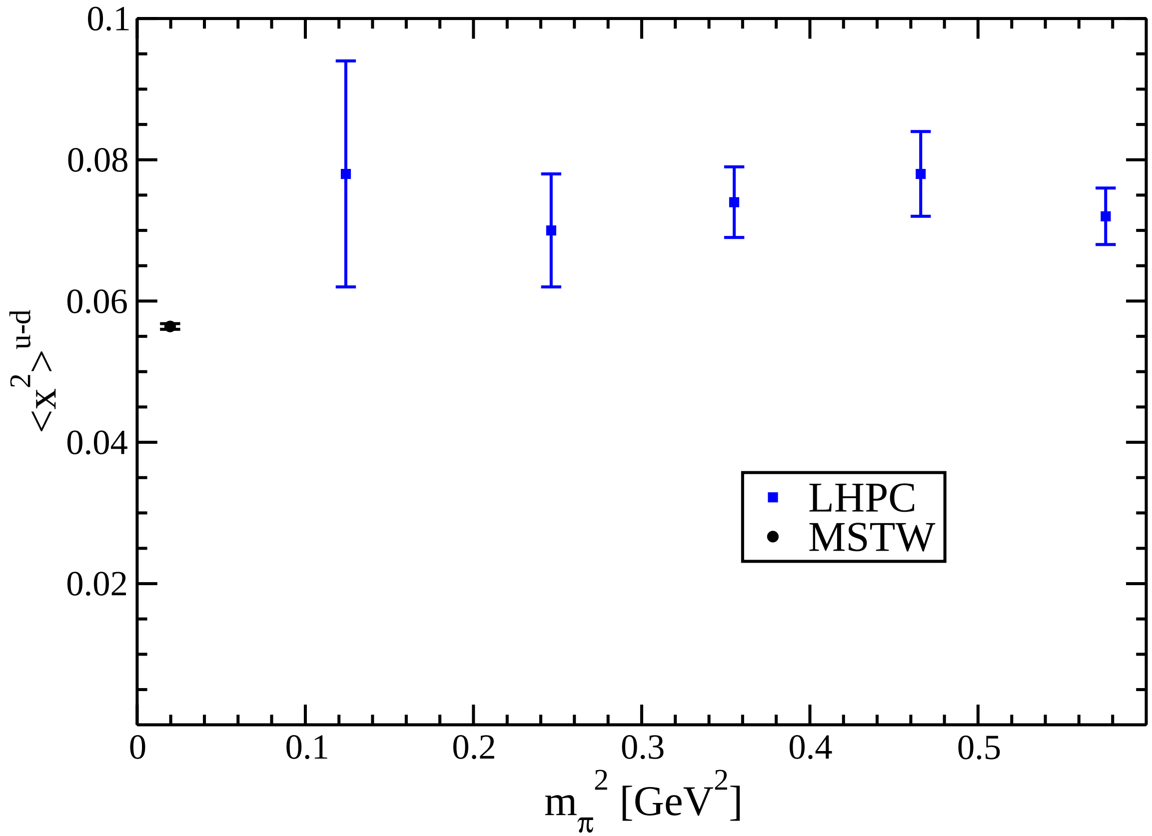


QCDSF LAT2010

- lack of any m_π dependence seems to preclude the use of chiral p.t.

Second moment

- calc. at a single lattice spacing shows near agreement in $\langle x^2 \rangle^{u-d}$



LHPC hep-lat/0705.4295

- mild cutoff effects could easily account for this small discrepancy

Outlook

- QCDSF $\langle x \rangle^{u-d}$ is showing a strikingly flat behavior down to almost the physical poin mass but overestimates the results from global fits

- other moments, g_A and $\langle x^2 \rangle$ shown as examples, are also very flat but the discrepancies are typically less severe than for $\langle x \rangle$

- in the near term, expect careful studies of $a \to 0$, $L \to \infty$ and $m_\pi \to 140$ MeV and other systematics (already underway)

- precise reproduction of well-measured nucleon properties will bolster confidence in a rich program of hadron structure from lattice QCD